Meservey-Tedrow-Fulde effect in a quantum dot embedded between metallic and superconducting electrodes

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Magnetic field applied to the quantum dot coupled between one metallic and one superconducting electrode can produce a similar effect as has been experimentally observed by Meservey *et al.* [Phys. Rev. Lett. **25**, 1270 (1970)] for the planar normal metal-superconductor junctions. We investigate the tunneling current and show that indeed the square-root singularities of differential conductance exhibit the Zeeman splitting near the gap-edge features $V=\pm\Delta/e$. Since magnetic field affects also the in-gap states of quantum dot, it furthermore imposes a hyperfine structure on the anomalous (subgap) Andreev current which has a crucial importance for a signature of the Kondo resonance.

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I. INTRODUCTION

Already in early days of the tunneling spectroscopy it has been shown that magnetic field B (which couples to spin of the charge carriers) is in superconductors responsible for splitting the square-root singularities of the tunneling conductance[1] by the Zeeman energy $2\mu_B B$, where μ_B is the Bohr magneton. This Meservey-Tedrow-Fulde (MTF) effect has been observed experimentally in the thin superconducting aluminum films applying parallel magnetic field so that orbital diamagnetic effects could be avoided. Similar qualitative results have been recently noticed in the measurements of c-axis tunneling for the layered high-temperature superconducting compounds.[2]

We argue that the MTF effect should be also feasible in various nanostructures consisting of a quantum dot (QD) placed between one metallic and one superconducting electrode. Zero-dimensional character of QDs in a natural way eliminates the influence of orbital effects therefore magnetic field would affect the charge transport only through the Zeeman term. This can in turn manifest itself in the differential conductance. Roughly speaking, the charge current flows if an external bias V exceeds the energy gap Δ (necessary to break the Cooper pairs into individual electrons) thereof the resulting conductance has a low-voltage onset near the gap edges $eV=\pm\Delta$. In the presence of a magnetic field, these gap-edge singularities are going to split (see Sec. III).

More detailed analysis of the charge tunneling[3] involves, however, also the additional (anomalous) channels due to mixing of the particle and hole excitations in superconductors. In particular, even at subgap voltages $|eV|\leq\Delta$ the mechanism of Andreev reflections provides a finite contribution to the conductance. Since the Andreev mechanism is very sensitive to location of the in-gap QD states[4–8] and the on-dot correlations,[9–18] we shall explore the influence of magnetic field on such subgap conductance. In Sec. IV we discuss a hyperfine structure for the Andreev conductance neglecting the correlations. In Sec. V we extend our study taking into account a finite value of the on-dot repulsion U. We show that appearance of the low-temperature Kondo resonance enhances the zero-bias conductance and this feature undergoes the Zeeman splitting when magnetic field is applied.

In practical terms, there have been considered some proposals for the magnetic field tuned Andreev scattering as an efficient cooling mechanism in two-dimensional electron gas—superconductor nanostructure[19] and a possibility to use the so-called Andreev quantum dot as a magnetic-flux detector.[20]

II. MODEL

For a general description of transport phenomena through a nanoscopic island placed between external leads one should consider a quantized multilevel structure of QD.[21] However, in the case when a level spacing is smaller in comparison to QD hybridization with the electrodes one can restrict to a simplified picture of the Anderson model[5,6,9–11]

$$\hat{H}=\hat{H}_N+\hat{H}_S+\sum_{\sigma}\epsilon_{d,\sigma}\hat{d}^{\dagger}_{\sigma}\hat{d}_{\sigma}+U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow} +\sum_{\mathbf{k},\sigma}\sum_{\beta=N,S}(V_{\mathbf{k}\beta}\hat{d}^{\dagger}_{\sigma}\hat{c}_{\mathbf{k}\sigma\beta}+V^{*}_{\mathbf{k}\beta}\hat{c}^{\dagger}_{\mathbf{k}\sigma,\beta}\hat{d}_{\sigma}). \quad (1)$$

Operators d_σ ($d^{\dagger}_{\sigma}$) denote the annihilation (creation) of electron whose energy level is $\varepsilon_{d,\sigma}$ and U is the on-dot Coulomb repulsion between opposite spin electrons. The last terms describe hybridization of QD with the normal ($\beta=N$) and superconducting ($\beta=S$) electrodes. Magnetic field eventually shifts the QD level by $\varepsilon_{d,\sigma}=\varepsilon_d-g_\sigma\mu_B B$, where the spin-dependent coefficients are defined as $g_\uparrow=1$ and $g_\downarrow=-1$.

Hamiltonian of the normal (metallic) lead is taken as $\hat{H}_N=\Sigma_{\mathbf{k},\sigma}\xi^{\sigma}_{\mathbf{k}N}\hat{c}^{\dagger}_{\mathbf{k}\sigma N}\hat{c}_{\mathbf{k}\sigma N}$ whereas for the superconducting electrode we choose the usual BCS form $\hat{H}_S=\Sigma_{\mathbf{k},\sigma}\xi^{\sigma}_{\mathbf{k}S}\hat{c}^{\dagger}_{\mathbf{k}\sigma S}\hat{c}_{\mathbf{k}\sigma S}-\Sigma_{\mathbf{k}}(\Delta\hat{c}^{\dagger}_{\mathbf{k}\uparrow S}\hat{c}^{\dagger}_{-\mathbf{k}\downarrow S}+\text{H.c.})$ with an isotropic energy gap Δ. The relative energies $\xi^{\sigma}_{\mathbf{k}\beta}=(\varepsilon_{\mathbf{k}\beta}-g_\sigma\mu_B B)-\mu_\beta$ are measured from the chemical potentials μ_β. We shall focus on the wide band limit $|V_{\mathbf{k}\beta}|\ll D$ (where $-D\leq\varepsilon_{\mathbf{k}\beta}\leq D$) and consider a small external voltage V, which detunes the chemical potentials by $\mu_N-\mu_S=eV$ inducing the charge flow through N-QD-S junction. We assume $|eV|$ to be much smaller than level spacings typical for the realistic QDs (Ref. 21) so that applicability of the model (1) can be justified.

Let us start by establishing the QD Green's function in the equilibrium situation, i.e., for $V=0$. Fourier transform of the retarded Green's functions can be formally expressed by the Dyson equation

$$\boldsymbol{G}_\sigma(\omega)^{-1} \equiv \begin{bmatrix} \langle\langle \hat{d}_\sigma;\hat{d}_\sigma^\dagger\rangle\rangle_\omega & \langle\langle \hat{d}_\sigma;\hat{d}_\sigma\rangle\rangle_\omega \\ \langle\langle \hat{d}_{-\sigma}^\dagger;\hat{d}_\sigma^\dagger\rangle\rangle_\omega & \langle\langle \hat{d}_{-\sigma}^\dagger;\hat{d}_{-\sigma}\rangle\rangle_\omega \end{bmatrix}^{-1} = \begin{bmatrix} \omega-\varepsilon_{d,\sigma} & 0 \\ 0 & \omega+\varepsilon_{d,-\sigma} \end{bmatrix} - \boldsymbol{\Sigma}_{d,\sigma}^0(\omega) - \boldsymbol{\Sigma}_{d,\sigma}^U(\omega), \tag{2}$$

where $\boldsymbol{\Sigma}_{d,\sigma}^0$ denotes the self-energy of noninteracting QD ($U=0$) and $\boldsymbol{\Sigma}_{d,\sigma}^U$ accounts for the correlation effects. For a simple understanding of the MTF effect it would be helpful to focus first on the uncorrelated QD when the self-energy is known exactly. Further corrections due to $\boldsymbol{\Sigma}_{d,\sigma}^U$ contribute a renormalization of the spectral function[14] whose impact on the charge transport will be discussed separately in Sec. V.

For convenience we introduce the hybridization coupling $\Gamma_\beta \equiv 2\pi\Sigma_{\mathbf{k}}|V_{\mathbf{k}\beta}|^2\delta(\omega-\varepsilon_{\mathbf{k}\beta})$ and define the following spin-dependent energy $\tilde{\omega}_\sigma=\omega+g_\sigma\mu_B B$. Imaginary part of the self-energy $\boldsymbol{\Sigma}_{d,\sigma}^0$ for $|\tilde{\omega}_\sigma|\le\Delta$ is given by $\mathrm{Im}\,\boldsymbol{\Sigma}_{d,\sigma}^0(\omega)=-\frac{\Gamma_N}{2}\mathbf{1}$ while at large energies $D>|\tilde{\omega}_\sigma|>\Delta$ it takes the following form:[12,15]

$$\mathrm{Im}\,\boldsymbol{\Sigma}_{d,\sigma}^0(\omega) = -\frac{1}{2}\begin{bmatrix} \Gamma_N+\Gamma_S\frac{|\tilde{\omega}_\sigma|}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}} & \Gamma_S\frac{\Delta\,\mathrm{sgn}(\tilde{\omega}_\sigma)}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}} \\ \Gamma_S\frac{\Delta\,\mathrm{sgn}(\tilde{\omega}_\sigma)}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}} & \Gamma_N+\Gamma_S\frac{|\tilde{\omega}_\sigma|}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}} \end{bmatrix}. \tag{3}$$

The corresponding real parts can be determined using the Kramers-Krönig relations.

Imaginary part of the self-energy $\boldsymbol{\Sigma}_{d,\sigma}^0$ has thus the square-root singularities at energies $\omega=\pm\Delta\pm\mu_B B$, so in presence of magnetic field there are altogether four such points. They show up as kinks in the spectral function $\rho_d(\omega)=\Sigma_\sigma\rho_{d,\sigma}(\omega)$, where

$$\rho_{d,\sigma}(\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\langle \hat{d}_\sigma;\hat{d}_\sigma^\dagger\rangle\rangle_{\omega+i0^+}. \tag{4}$$

We shall see below that appearance of such characteristic points leads to the MTF effect observed in the tunneling conductance.

III. MESERVEY-TEDROW-FULDE EFFECT

To compute the tunneling current we adopt the formalism outlined in the previous studies[8,9,12] extending it here on a situation with the spin sensitive transport due to magnetic field. The steady charge current is defined as $I(V) = -e\frac{d}{dt}\langle\Sigma_{\mathbf{k},\sigma}\hat{c}_{\mathbf{k}\sigma N}^\dagger\hat{c}_{\mathbf{k}\sigma N}\rangle = e\frac{d}{dt}\langle\Sigma_{\mathbf{k},\sigma}\hat{c}_{\mathbf{k}\sigma S}^\dagger\hat{c}_{\mathbf{k}\sigma S}\rangle$. We carry out the time derivative and determine the expectation value using the nonequilibrium Keldysh Green's functions.

In analogy to the standard Blonder-Tinkham-Klawijk theory[3] we express the current as composed of two contributions

$$I(V) = I_1(V) + I_A(V). \tag{5}$$

The first part $I_1(V)$ stands for a contribution which at low temperatures appears practically outside the energy gap $|eV|\ge\Delta$. Its magnitude is expressed by the Landauer-type formula

$$I_1(V) = \frac{e}{h}\sum_\sigma\int d\omega T_{1,\sigma}(\omega)[f(\omega+eV)-f(\omega)], \tag{6}$$

where $f(\omega)=[1+\exp(\omega/k_BT)]^{-1}$. The transmittance $T_{1,\sigma}(\omega)$ is nonvanishing only outside the energy gap $|\tilde{\omega}_\sigma|\ge\Delta$ and is given by the following parts of the retarded Green's functions[8,12]

$$T_{1,\sigma}(\omega) = \frac{\Gamma_N\Gamma_S|\tilde{\omega}_\sigma|}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}}(|\langle\langle \hat{d}_\sigma;\hat{d}_\sigma^\dagger\rangle\rangle_\omega|^2+|\langle\langle \hat{d}_\sigma;\hat{d}_{-\sigma}\rangle\rangle_\omega|^2) - \frac{2\Gamma_N\Gamma_S\Delta}{\sqrt{\tilde{\omega}_\sigma^2-\Delta^2}}\mathrm{Re}\{\langle\langle \hat{d}_\sigma;\hat{d}_\sigma^\dagger\rangle\rangle_\omega\langle\langle \hat{d}_\sigma;\hat{d}_{-\sigma}\rangle\rangle_\omega^*\}. \tag{7}$$

The second part in Eq. (5) originates from the mechanism of Andreev reflections[3,9,12]

$$I_A(V) = \frac{e}{h}\sum_\sigma\int d\omega T_{A,\sigma}(\omega)[f(\omega+eV)-f(\omega-eV)]. \tag{8}$$

Its transmittance is finite even inside the energy gap[8,12]

$$T_{A,\sigma}(\omega) = \Gamma_N^2|\langle\langle \hat{d}_\sigma;\hat{d}_{-\sigma}\rangle\rangle_\omega|^2. \tag{9}$$

Physically such process occurs when an incident electron from N electrode (of arbitrary energy) is converted into a pair on QD (with a simultaneous reflection of a hole) and it propagates in S electrode as a Cooper pair. This anomalous Andreev current is closely related to the off-diagonal order parameter induced in the QD (proximity effect).[14,15]

Figure 1 illustrates the influence of magnetic field on the total differential conductance $G(V)=\frac{d}{dV}I(V)$ obtained for N-QD-S junction. We clearly notice the Zeeman splitting of the square-root singularities resembling the former experimental observation for N-I-S (I-insulator) junction.[1] However, in a present case the conductance does not saturate to a finite value far outside the gap $|eV|\gg\Delta$ because the QD spectrum spreads only nearby ε_d [usually in realistic multilevel QDs there would be seen the quantum oscillations of $G(V)$ (Ref. 21)]. The in-gap features related to the Andreev current are discussed in Secs. IV and V.

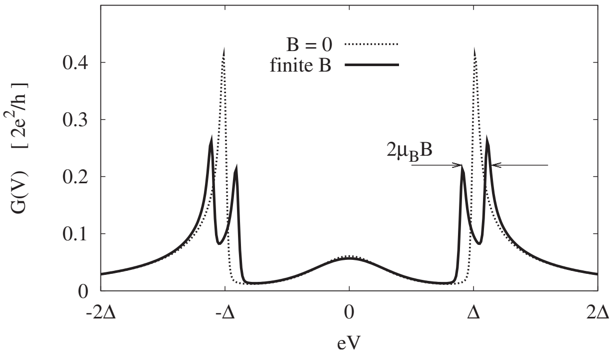

FIG. 1. The differential conductance $G(V)$ versus bias voltage V for N-QD-S junction. Notice a splitting of the gap-edge singularities around $eV=\pm\Delta$ induced by magnetic field B. We used for computations $\varepsilon_d=0$, $U=0$, $\Gamma_N=\Delta$, $\Gamma_S=0.1\Delta$, $T=0.01\Delta$ assuming $\Delta=0.1D$.

IV. MAGNETIC-FIELD EFFECT ON THE ANDREEV CURRENT

The mechanism of Andreev reflections transmits the charge current even for the subgap voltages. To focus solely on this anomalous current it is convenient to consider the extreme limit $\Delta\to\infty$ as proposed by Tanaka *et al.*[14] In such case I_1 can be completely discarded from our analysis. Using Eq. (3) we obtain the self-energy $\mathbf{\Sigma}^0_{d,\sigma}$ simplified to[9,14,15]

$$\mathbf{\Sigma}^0_{d,\sigma}(\omega) = -\frac{1}{2}\begin{bmatrix} i\Gamma_N & \Gamma_S \\ \Gamma_S & i\Gamma_N \end{bmatrix}. \tag{10}$$

Upon neglecting the Coulomb correlations one can analytically determine Green's function (2), where the spin-dependent spectral function (4) acquires the BCS structure[14]

$$\rho_{d,\sigma}(\omega) = \frac{1}{2}\left[1+\frac{\varepsilon_d}{E_d}\right]\frac{\frac{1}{\pi}\Gamma_N/2}{(\tilde{\omega}_\sigma - E_d)^2 + (\Gamma_N/2)^2} + \frac{1}{2}\left[1-\frac{\varepsilon_d}{E_d}\right]\frac{\frac{1}{\pi}\Gamma_N/2}{(\tilde{\omega}_\sigma + E_d)^2 + (\Gamma_N/2)^2} \tag{11}$$

with a quasiparticle energy $E_d=\sqrt{\varepsilon_d^2+(\Gamma_S/2)^2}$. The in-gap QD states (often referred as *Andreev bound states*) form around $\pm E_d\pm\mu_B B$ as illustrated in the upper panel of Fig. 2. Their line broadening is given by $\Gamma_N/2$ and in absence of magnetic field the particle-hole splitting is controlled by Γ_S (Refs. 14 and 15) (the dashed line in Fig. 2). Magnetic field further enforces the Zeeman splitting of these in-gap states.

Above mentioned behavior has an indirect effect on the off-diagonal parts of Green's function (2) which in turn determine the Andreev transmittance. In the limit $\Delta\to\infty$ Eq. (9) reduces to

$$T_{A,\sigma}(\omega) = \frac{\Gamma_N^2(\Gamma_S/2)^2}{[(\tilde{\omega}_\sigma - E_d)^2 + (\Gamma_N/2)^2][(\tilde{\omega}_\sigma + E_d)^2 + (\Gamma_N/2)^2]}. \tag{12}$$

The subgap Andreev conductance $G_A(V)=\frac{d}{dV}I_A(V)$ is thus characterized by a four peak structure as shown in the bottom panel of Fig. 2. Obviously the weights of particle and hole peaks of the spectral function (11) as well as their weights in the Andreev transmittance [Eq. (12)] depend on the QD level ε_d. Variation in the Andreev conductance with respect to (V,ε_d) is plotted in Fig. 3. We can notice that optimal conditions for the subgap current occur when the QD level is located near the energy-gap center, otherwise the proximity effect is less efficient.

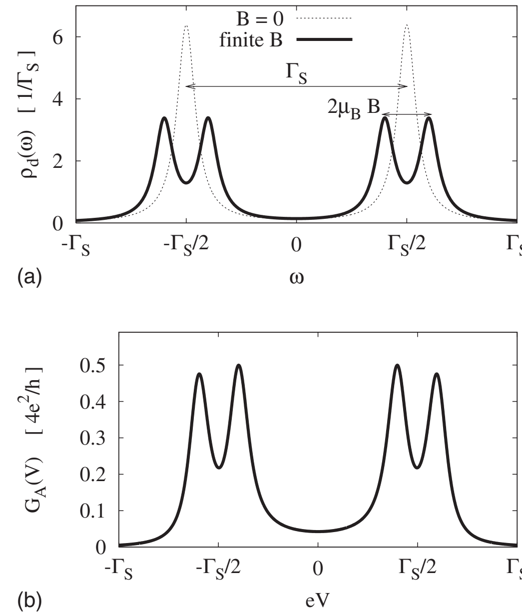

FIG. 2. Zeeman splitting of the bound Andreev states for the QD located in the center of superconducting gap $\varepsilon_d=0$. Upper panel illustrates the density of states $\rho_d(\omega)$ and the bottom figure shows differential conductance of the in-gap current. For computations we used $\Gamma_N=0.1\Gamma_S$, $\mu_B B=0.1\Gamma_S$ assuming $\Gamma_S=0.01D$ and $U=0$.

On top of the particle-hole structure seen in the Andreev states there is an additional Zeeman splitting brought by magnetic field. In Fig. 4 we sketch the Andreev conductance in (V,B) plane for $\varepsilon_d=0$, where the dark areas correspond to a maximal value $4e^2/h$. There appears a characteristic diamond shape marking the positions of such maximal conductance $G_A(V,B)$. We believe that this hyperfine structure could be probed experimentally.

To complete the discussion of the subgap Andreev current we briefly comment on a possible influence of an asymmetry

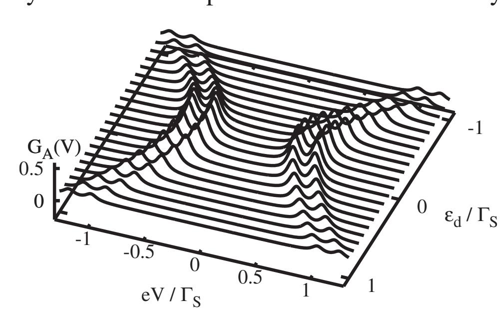

FIG. 3. Differential conductance $G_A(V)$ of the in-gap Andreev current as a function of the bias voltage V and the QD level ε_d. We used for computations $\Gamma_S=0.01D$, $\Gamma_N=0.1\Gamma_S$, and $T=0.01\Gamma_S$, and set the magnetic field $\frac{1}{2}\mu_B B=0.1\Gamma_S$. The conductance is expressed in units of $4e^2/h$.

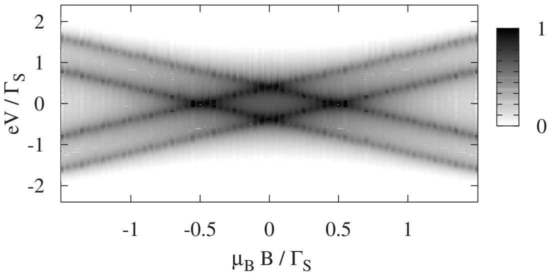

FIG. 4. Differential conductance $G_A(V)$ of the in-gap Andreev current as a function of the bias voltage V and magnetic field B for the QD level $\varepsilon_d=0$ and $\Gamma_N=0.1\Gamma_S$, $T=0.01\Gamma_S$. Dark areas denote the regions where G_A approaches the value $4e^2/h$.

between the hybridization couplings Γ_N and Γ_S. We explore for this purpose the zero-bias conductance $G_A(V=0)$. At low temperature we find from Eq. (12) that

$$G_A(0) = \frac{4e^2}{h}\frac{\Gamma_N^2(\Gamma_S/2)^2}{\left[(\mu_B B - E_d)^2 + \left(\frac{\Gamma_N}{2}\right)^2\right]\left[(\mu_B B + E_d)^2 + \left(\frac{\Gamma_N}{2}\right)^2\right]}. \tag{13}$$

In Fig. 5 we show the influence of magnetic field on the zero-bias Andreev conductance for several values of the asymmetry rate Γ_N/Γ_S. If $\Gamma_N/\Gamma_S \ll 1$ then a line broadening of the Andreev states diminishes so in consequence the particle and hole peaks become well separated. Under such conditions the subgap conductance has maxima around the quasiparticle states at $\pm\Gamma_S/2$ (where the ideal conductance $4e^2/h$ is reached). Let us recall that in absence of magnetic field, Eq. (13) reproduces for $\varepsilon_d=0$ the well-known result $G_A(0)=\frac{4e^2}{h}(\frac{2\Gamma_N\Gamma_S}{\Gamma_S^2+\Gamma_N^2})^2$.[14] For the symmetric coupling $\Gamma_S=\Gamma_N$ it yields $G_A(0)=4e^2/h$.[9]

V. INFLUENCE OF THE COULOMB CORRELATIONS

In the limit $\Delta\to\infty$ the self-energy $\mathbf{\Sigma}^0_{d,\sigma}$ becomes a static quantity [Eq. (10)], therefore the role of superconducting lead can be exactly replaced by the on-dot gap parameter $\Delta_d=\Gamma_S/2$. Instead of Eq. (1) we can thus use the following auxiliary Hamiltonian

$$\hat{H} = \hat{H}_N + \sum_{\mathbf{k},\sigma}(V_{\mathbf{k}N}\hat{d}^\dagger_\sigma\hat{c}_{\mathbf{k}\sigma\beta} + \text{h.c.}) + \sum_\sigma \epsilon_{d,\sigma}\hat{d}^\dagger_\sigma\hat{d}_\sigma + (\Delta_d\hat{d}^\dagger_\uparrow\hat{d}^\dagger_\downarrow + \text{H.c.}) + U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow}, \tag{14}$$

which turns out to be very convenient for investigating the correlations. Tanaka and co-workers[13,14] were able to rigorously prove that the self-energy $\mathbf{\Sigma}^U_{d,\sigma}$ must have a diagonal structure due to invariance of $U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow}$ term on the Bogoliubov-Valatin transformation.

In the remaining part of this section we shall focus on the subgap Andreev current transmitted through the correlated QD. The matrix Green's function (2) simplifies in the limit $\Delta\to\infty$ to the following (exact) structure:

$$\mathbf{G}_\sigma(\omega) = \begin{pmatrix} \omega - \varepsilon_{d,\sigma} - \Sigma_{N,\sigma}(\omega) & \frac{1}{2}\Gamma_S \\ \frac{1}{2}\Gamma_S & \omega + \varepsilon_{d,-\sigma} + \Sigma^*_{N,-\sigma}(-\omega) \end{pmatrix}^{-1}. \tag{15}$$

Influence of the correlations have been so far analyzed for the Hamiltonian (1) using various techniques.[9–15] Here we estimate the diagonal self-energy $\Sigma_{N,\sigma}(\omega)$ within Eq. (14) by the equation of motion method[22,23]

$$\omega - \varepsilon_{d,\sigma} - \Sigma_{N,\sigma}(\omega) = \frac{[\omega - \varepsilon_{d,\sigma} - \Sigma^0_{d,\sigma}(\omega)][\omega - \varepsilon_{d,\sigma} - U - \Sigma^0_{d,\sigma}(\omega) - \Sigma^3_{d,\sigma}(\omega)] + U\Sigma^1_{d,\sigma}(\omega)}{\omega - \varepsilon_{d,\sigma} - \Sigma^0_{d,\sigma}(\omega) - \Sigma^3_{d,\sigma}(\omega) - U[1 - \langle\hat{n}_{d,-\sigma}\rangle]}, \tag{16}$$

where $\Sigma^{\nu=1,3}_{d,\sigma}(\omega)$ are given by[22]

$$\Sigma^\nu_{d,\sigma}(\omega) = \sum_{\mathbf{k}}|V_{\mathbf{k}N}|^2\left(\frac{1}{\omega + \xi_{\mathbf{k}N} - \varepsilon_{d,-\sigma} - \varepsilon_{d,\sigma} - U} + \frac{1}{\omega - \xi_{\mathbf{k}N} + \varepsilon_{d,-\sigma} - \varepsilon_{d,\sigma}}\right)[f(\omega,T)]^{3-\nu/2}. \tag{17}$$

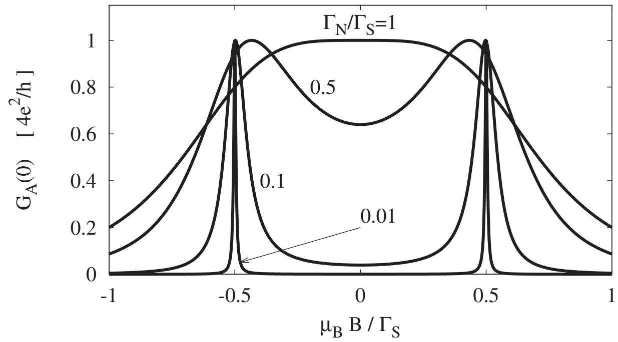

FIG. 5. The zero-bias differential conductance $G_A(0)$ as a function the magnetic field B for several relative values of Γ_N/Γ_S. We used for computations $\varepsilon_d=0$ assuming $T\to0$.

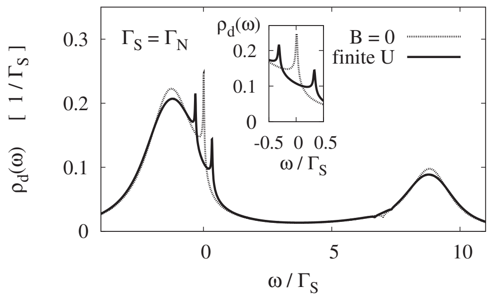

FIG. 6. Spectral function of the correlated QD obtained for $\varepsilon_d = -1.5\Gamma_S$, $U=10\Gamma_S$, $\Gamma_N=\Gamma_S$ and temperature $T=10^{-3}\Gamma_S$ ($\ll T_K$) in the limit $\Delta\to\infty$. Solid line corresponds to $\mu_B B=\Gamma_S/3$.

Approximations (16) and (17) qualitatively reproduce the following properties caused by on-dot correlations: (i) the charging effect and (ii) a possible appearance of the Kondo resonance for temperatures smaller than $T_K = \frac{\sqrt{U\Gamma_N}}{2}\exp\{\pi\varepsilon_d(\varepsilon_d+U)/U\Gamma_N\}$. The latter one is related to screening of the quantum dot spin by itinerant electrons of the metallic lead. In the case when energy level ε_d is located slightly below μ_N the hybridization $V_{\mathbf{k}N}$ induces effectively antiferromagnetic interaction between the QD and metallic lead. In consequence the bound singlet state can be formed giving rise to the resonance at $\omega=\mu_N$ for temperatures $T \le T_K$. Magnetic field eventually splits this resonance as illustrated in Fig. 6.

Any features present in the QD spectrum are further showing up in the measurable differential conductance. This is also valid for the Kondo resonance. Since it forms near the chemical potential μ_N, therefore its signatures appear predominantly in the low-voltage current. In fact, it has been shown that Kondo resonance enhances at low temperatures the zero-bias Andreev conductance,[9,15] however, its magnitude remains much smaller than the unitary limit value $2e^2/h$ typical for N-QD-N systems in the Kondo regime. In the present context we emphasize that magnetic field enforces the Zeeman splitting of the zero-bias Andreev anomaly in much the same way as it affects the zero-bias anomaly for the QD coupled to both metallic leads.[24,25]

The zero-bias enhancement of the Andreev conductance is a feature whose presence might be difficult to notice[9,10,15] unless some stringent requirements are fulfilled.[23] It turns out that optimal conditions for the low-temperature enhancement of $G_A(V\sim 0)$ take place when Γ_S is comparable to Γ_N (see Figs. 7 and 8) and ε_d is located slightly below the energy-gap center. For an increasing asymmetry between the hybridizations Γ_N and Γ_S the magnitude of low voltage Andreev conductance diminishes (similarly as we have been shown in Sec. IV upon neglecting the correlations). On the other hand, for ε_d moving far aside from the superconductor's gap center the proximity effect becomes weakened and the overall Andreev conductance is again suppressed.

In general it seems that an interplay between the on-dot pairing (absorbed from the superconducting electrode) and the Kondo state (due to screening of QD spin by the metallic lead electrons) has the same character as a competition of superconductivity versus magnetism in the solid-state physics. Since this is outside the main scope of the present topic we shall discuss it separately.[23] A combination of the Kondo physics, superconductivity, and the Zeeman polarization is a complex problem, and to our knowledge only few papers have so far attempted to address this challenging issue.[26,27]

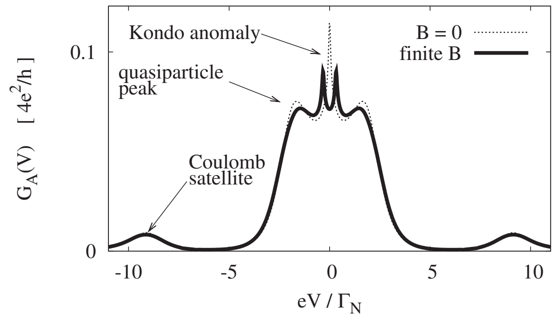

FIG. 7. Effect of magnetic field on the subgap Andreev conductance $G_A(V)$ in the Kondo regime with $\varepsilon_d=-1.5\Gamma_N$, $U=10\Gamma_N$, $\Gamma_S=5\Gamma_N$, and $T=10^{-3}\Gamma_N \ll T_K$. Notice appearance of: (i) the zero-bias Kondo anomaly (showing the Zeeman splitting for $\mu_B B=\Gamma_N/3$, (ii) the quasiparticle peaks at $|eV|\simeq E_d$, and (iii) Coulomb satellite peaks near $|eV|\simeq U$.

VI. SUMMARY

We have explored the effect of magnetic field on charge transport through the quantum dot attached to one normal and one superconducting electrode. For a bias voltage $V \simeq \pm\Delta/e$ we find the Zeeman splitting of the square-root singularities in the differential conductance. This resembles the experimental result of Meservey, Tedrow and Fulde observed in the N-I-S junction[1] which for the N-QD-S structures it seems rather easy to achieve.

We have extended our study also on the in-gap Andreev current. Due to the proximity effect the particles and holes of the quantum dot get mixed and effectively the spectrum acquires the BCS-like structure [Eq. (11)]. Differential conductance $G_A(V)$ of the in-gap current indirectly probes such structure of the bound Andreev states. We have shown that magnetic field leads to appearance of four peaks via the combined particle-hole and Zeeman splittings. We hope that this result might stimulate a search for the experimental detection of above mentioned structures.

Moreover, we have explored influence of the on-dot Coulomb interactions on the subgap Andreev current assuming

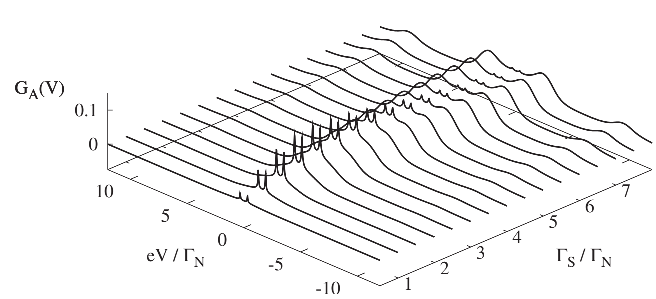

FIG. 8. The differential Andreev conductance $G_A(V)$ (in units $4e^2/h$) as a function of the bias voltage V and the asymmetry ratio Γ_S/Γ_N. We used the same set of parameters as in Fig. 7.

the extreme limit $\Delta \rightarrow \infty$. In general, the on-dot correlations contribute to the following QD spectrum: (i) appearance of the Coulomb satellite near $\omega = \varepsilon_{d,\downarrow} + U$ (charging effect) and (ii) at sufficiently low temperatures can produce the narrow Kondo resonance at the chemical potential μ_N. Magnetic field imposes the hyperfine splitting onto such spectrum in a similar way as has been observed in N-QD-N junctions.[25] The Kondo effect alone is exemplified in the zero-bias Andreev conductance where under appropriate conditions,[23] a low-temperature enhancement can be seen if $\Gamma_N \sim \Gamma_S$ and the gate voltage tunes ε_d nearly to the energy-gap center.

It would be of interest to use some more sophisticated methods for treating the on-dot interaction U in order to check whether there exists a minimal magnetic field necessary for splitting the Kondo peak (as theoretically predicted for N-QD-N junctions[24]) observable in the Andreev conductance. One can also study QD coupled with d-wave superconductor, where the square-root singularities are replaced by weaker kinks. We think that the Meservey-Tedrow-Fulde effect would be observable there too (but in a less pronounced manner) whereas the subgap conductance might qualitatively change.

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